Regression and its Applications

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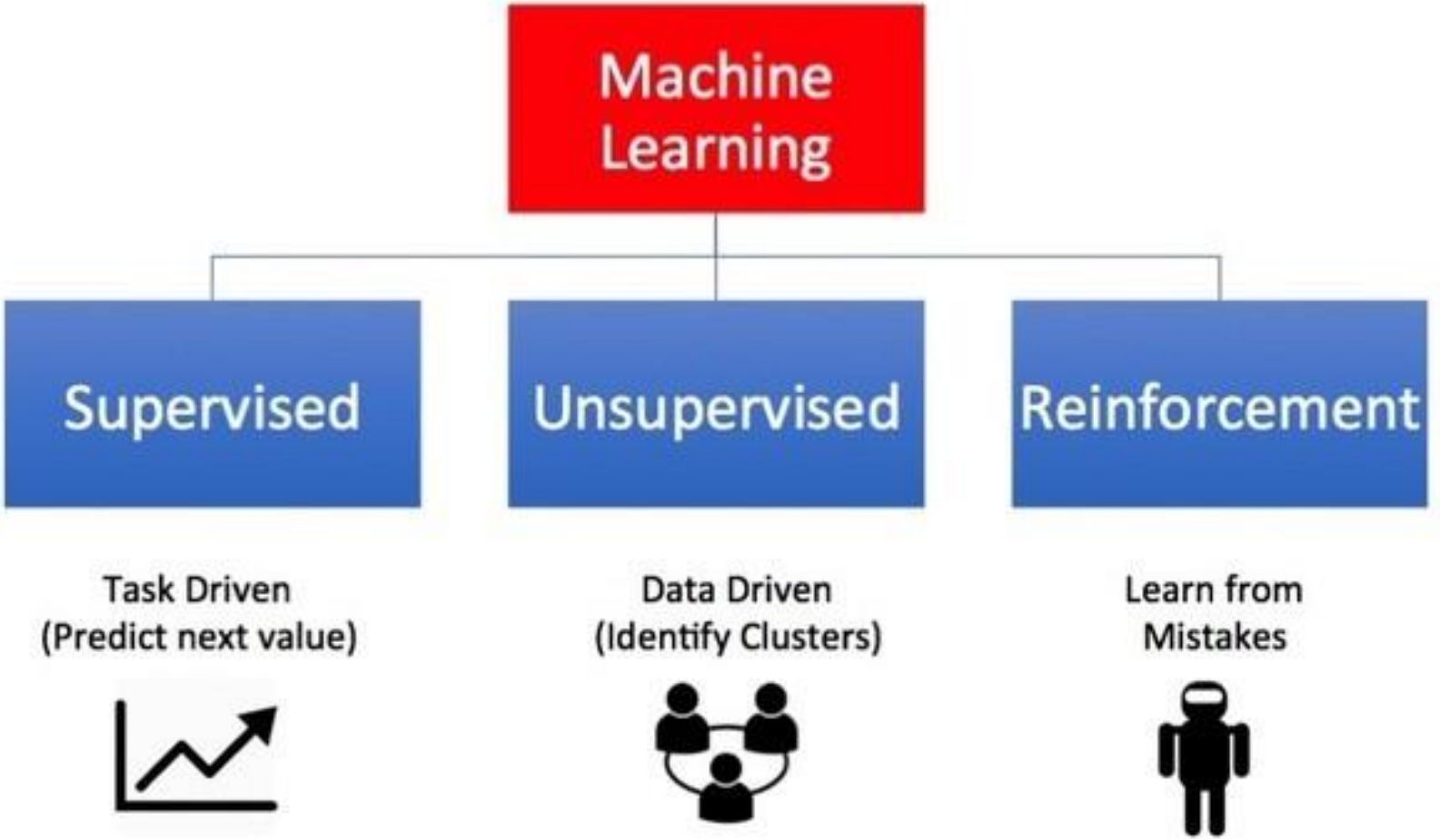
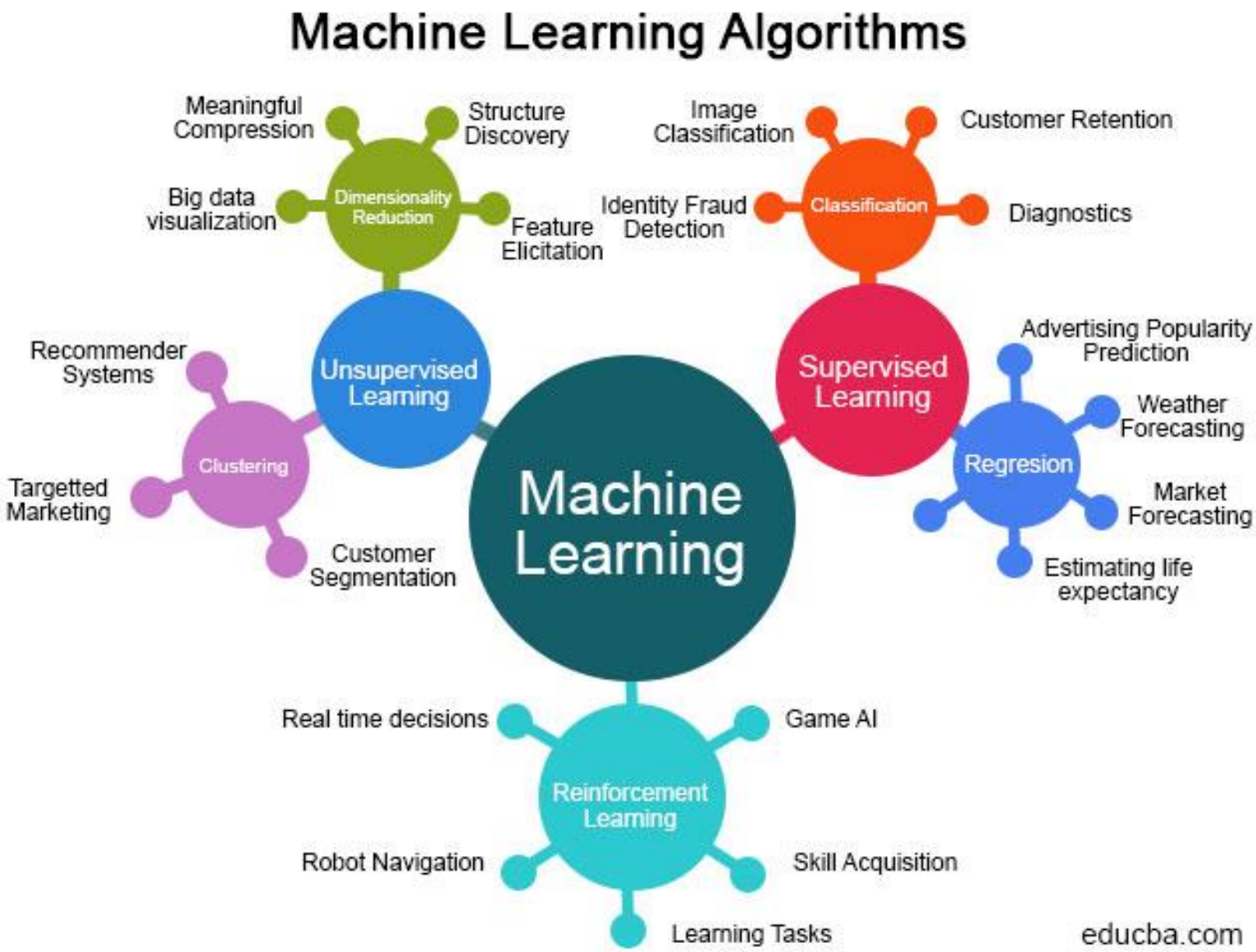
Types of Machine Learning

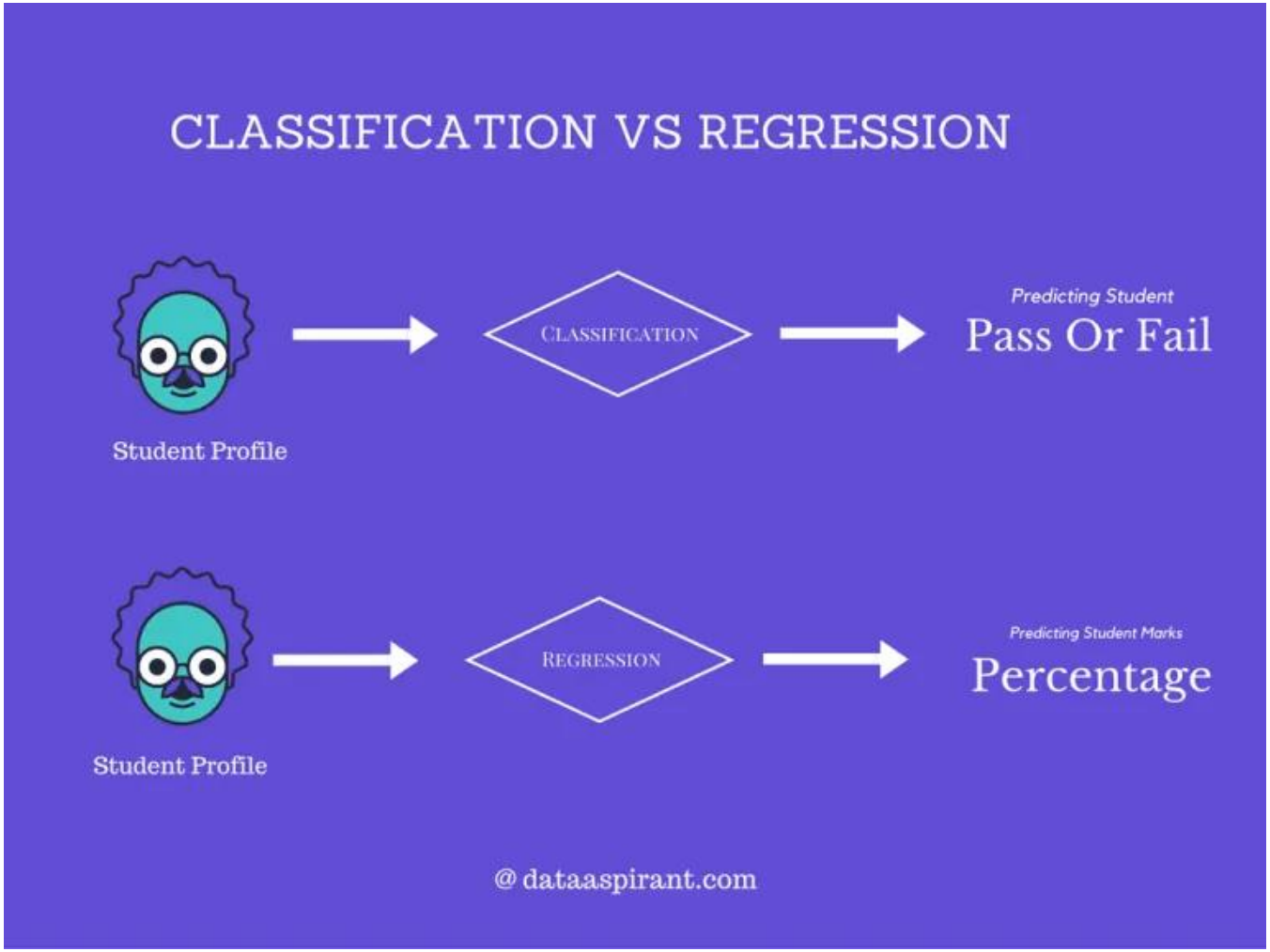


https://towardsdatascience.com/what-are-the-types-of-machine-learning-e2b9e5d1756f



https://www.educba.com/machine-learning-algorithms/

Classification vs. Regression

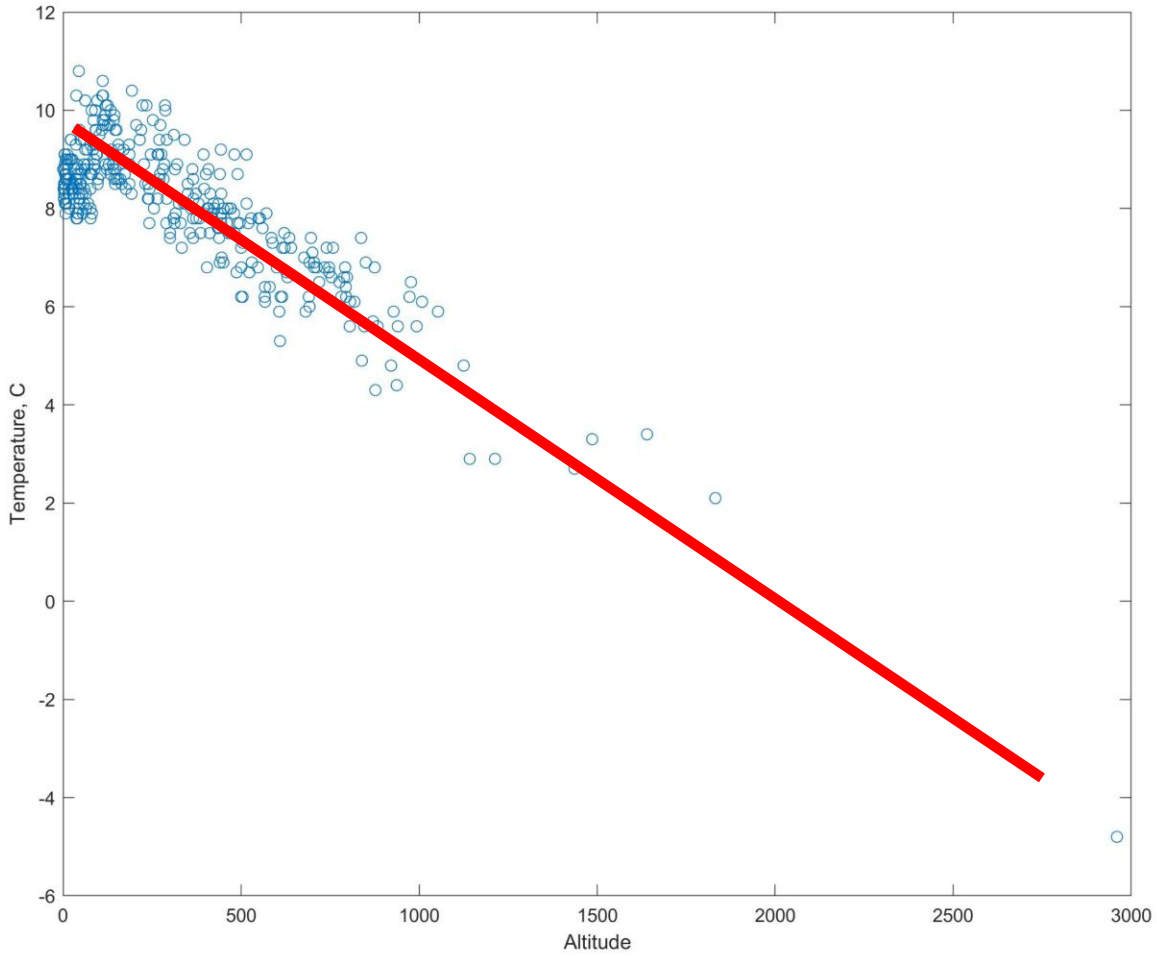


https://dataaspirant.com/2014/09/27/classification-and-prediction/

What is Linear Regression?

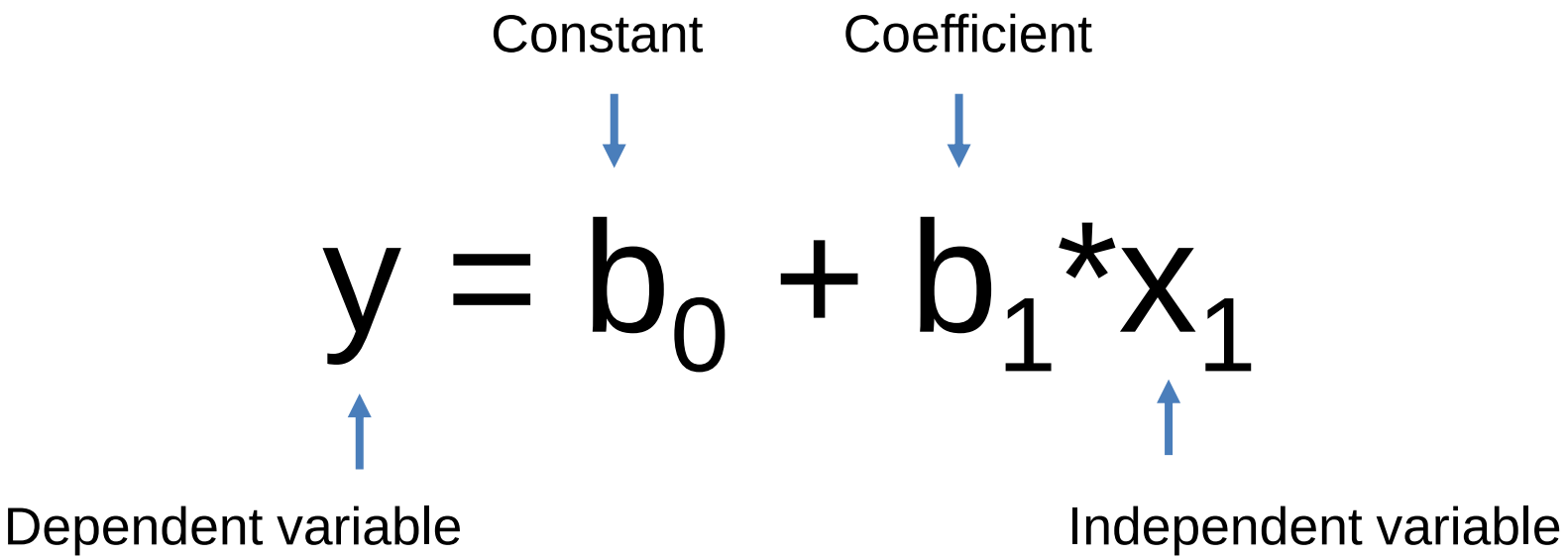
- **Learning**
  - A supervised algorithm that learns from a set of training samples.
  - Each training sample has one or more input values and a single output value.
  - The algorithm learns the line, plane or hyper-plane that best fits the training samples
- **Prediction**
  - Use the learned line, plane or hyper-plane to predict the output value for any
  - Input sample

Altitude vs. Temperature

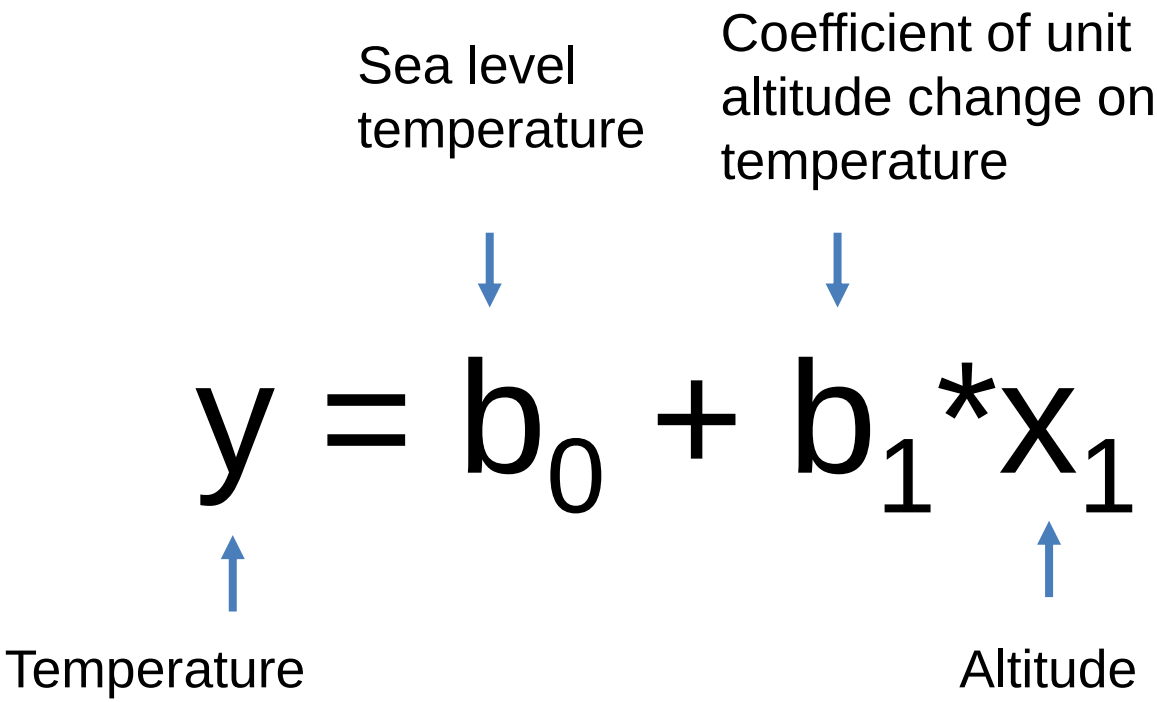


DWD data (Deutscher Wetterdienst)
data was taken at 349 weather station

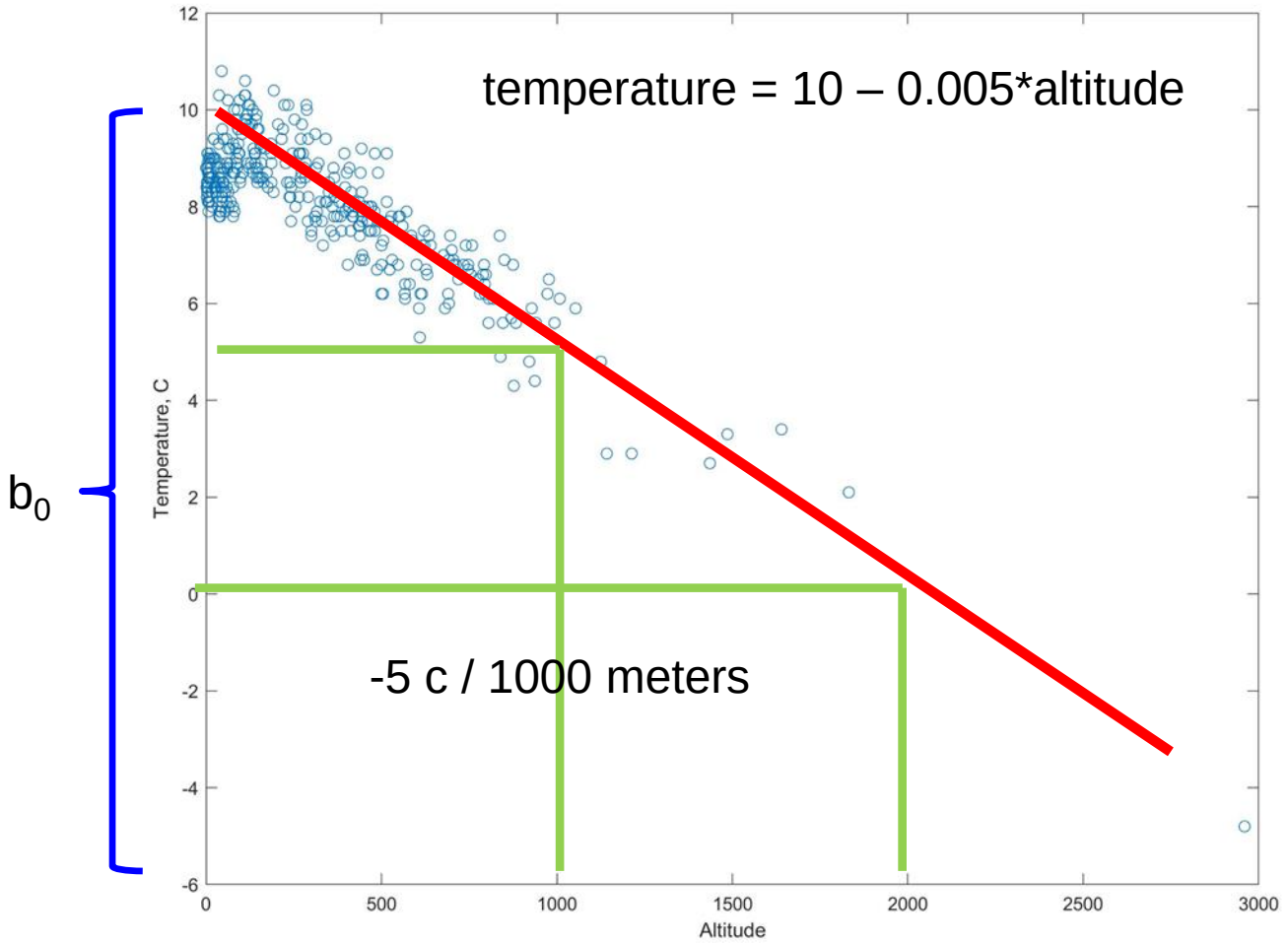
Simple Linear Regression



Simple Linear Regression



Altitude vs. Temperature



DWD data (Deutscher Wetterdienst)
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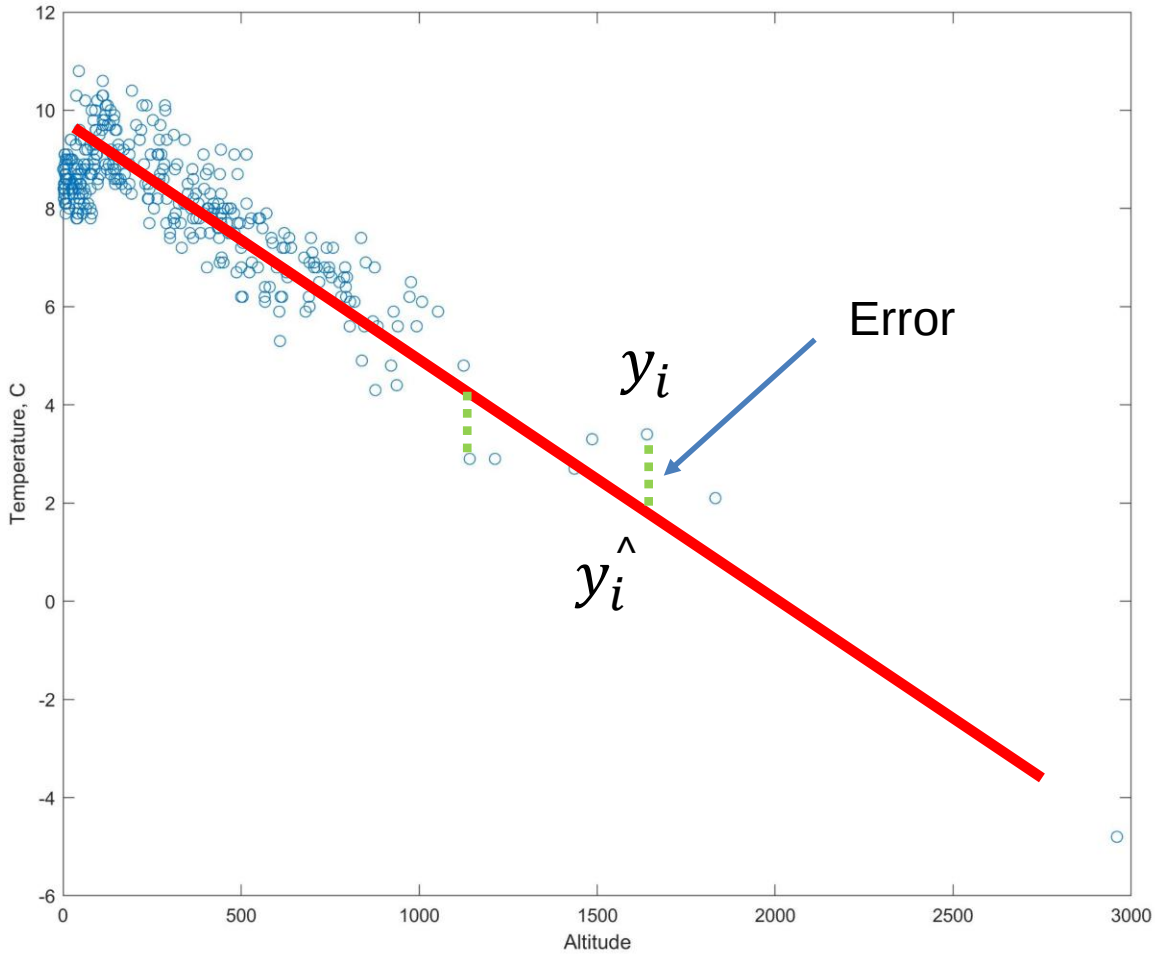
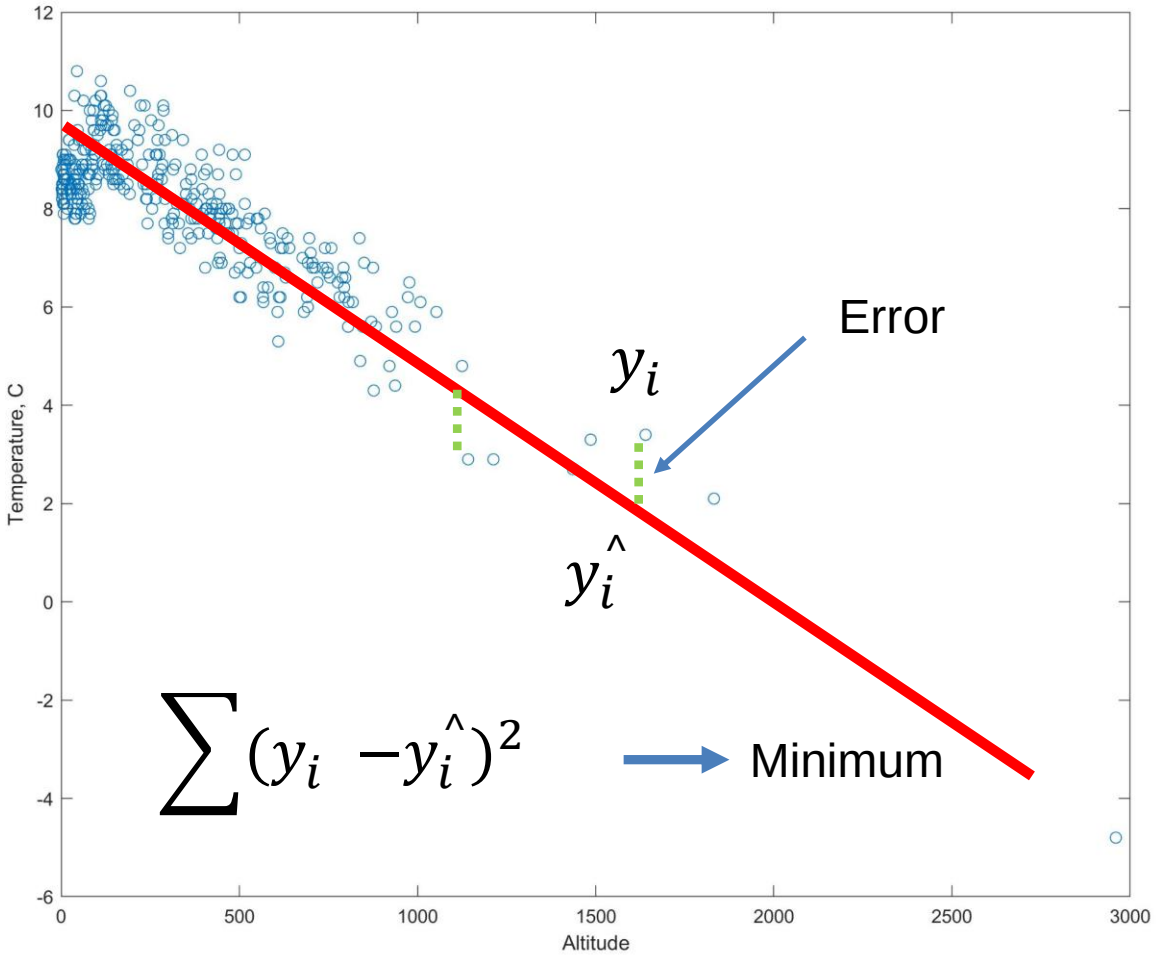
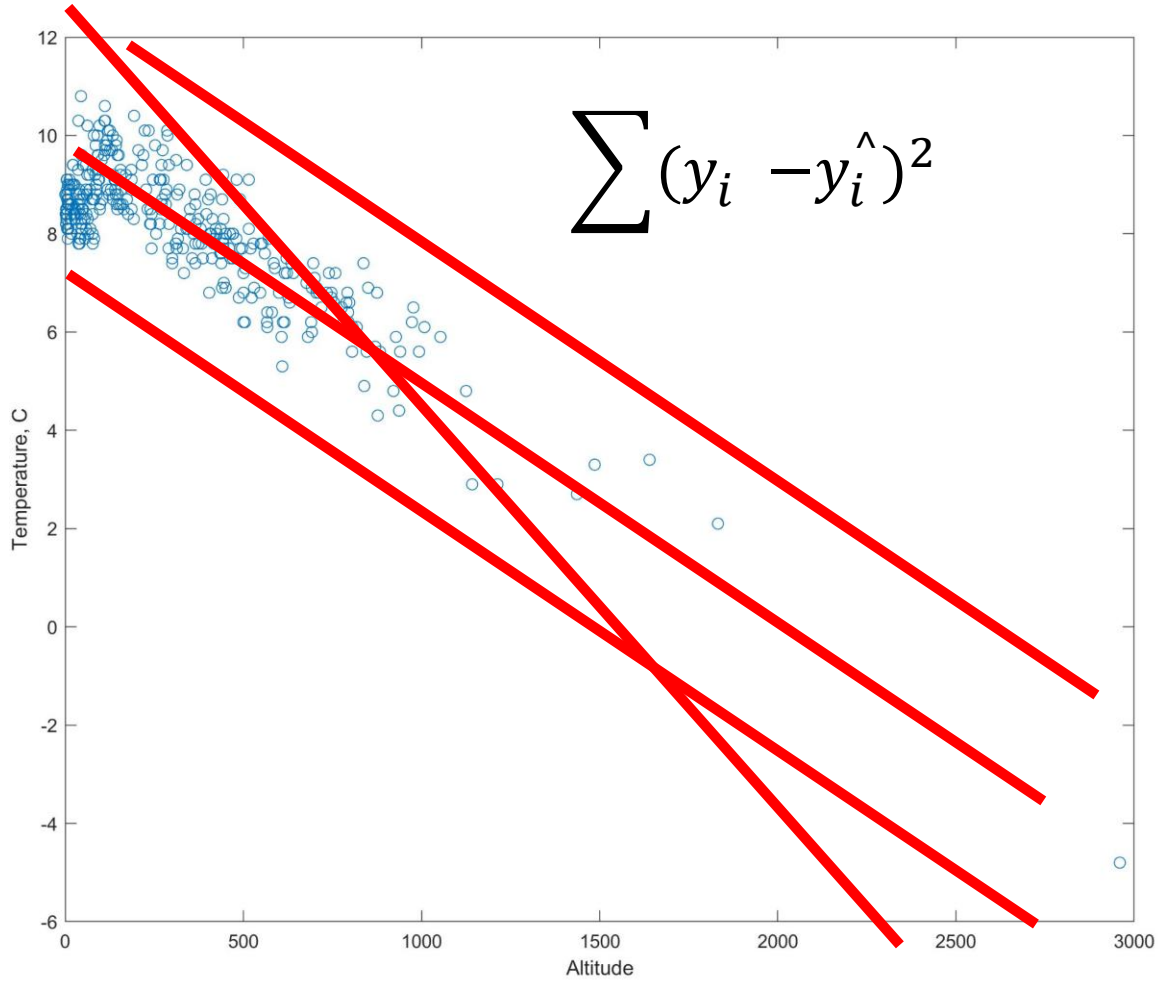
How to find errors?



How to define errors?
Ordinary Least Square



How to find the best fitted line?



Simple vs. Multiple Linear Regression

- Simple Linear Regression

$$y = b_0 + b_1 * x_1$$

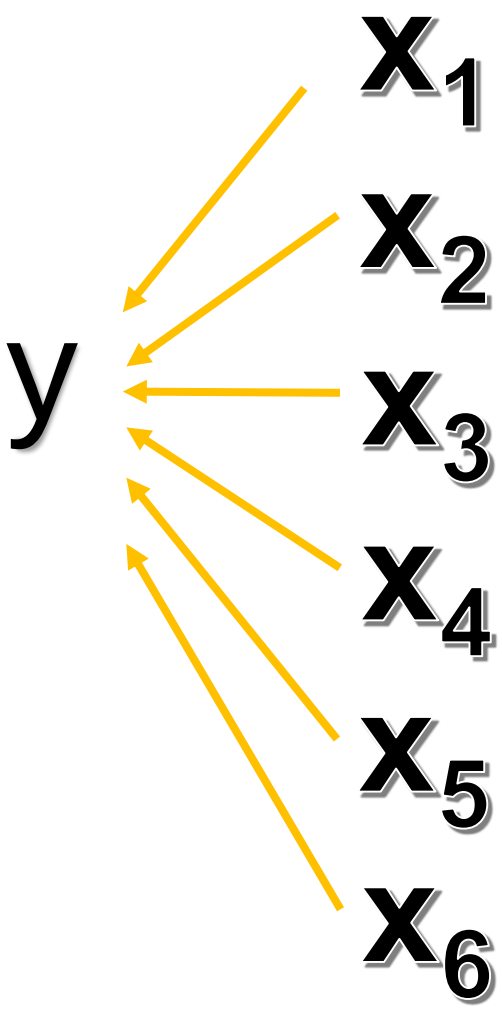
- Multiple Linear Regression

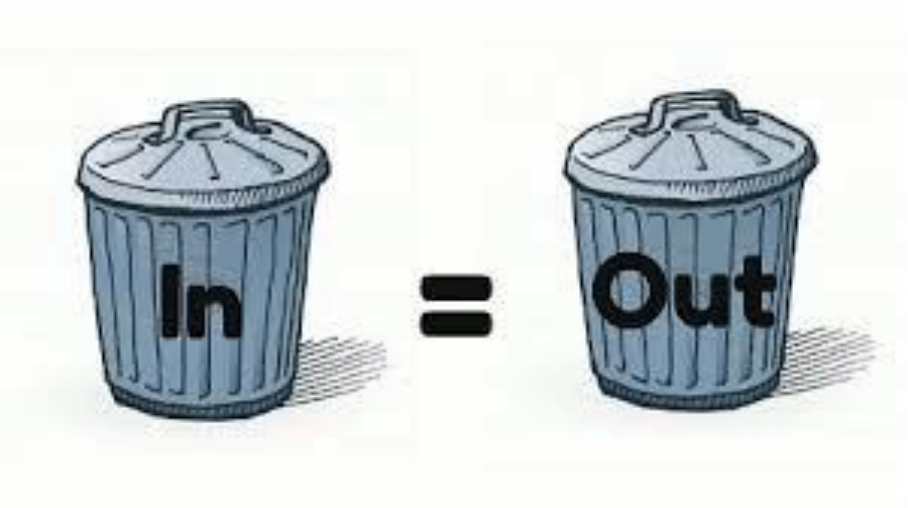
$$y = b_0 + b_1 * x_1 + b_2 * x_2 + b_3 * x_3 + \ldots + b_n * x_n$$

Assumption of Linear Regression

- Linearity
- Homoscedasticity
- Multivariate normality
- Independence of errors
- Lack of multicollinearity

Building a Multiple Regression Model



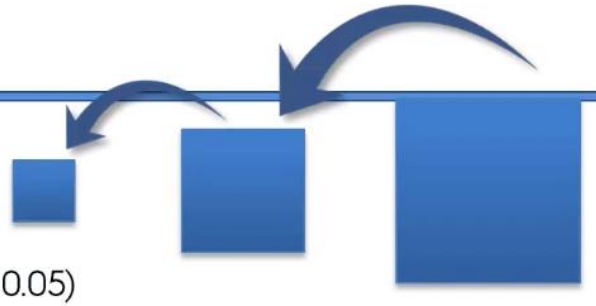


Variable Selection Methods

- All-in
- Backward Elimination
- Forward Selection
- Bidirectional Elimination
- Score comparison

Stepwise Method

Building A Model



Backward Elimination

STEP 1: Select a significance level to stay in the model (e.g. SL = 0.05)

STEP 2: Fit the full model with all possible predictors

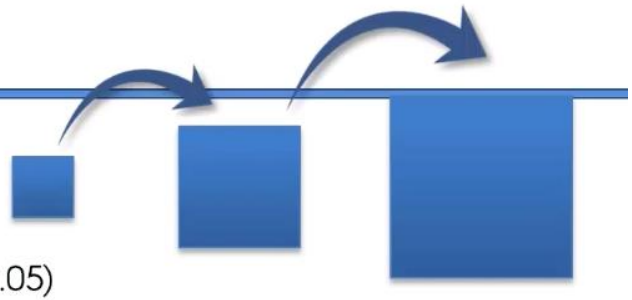
STEP 3: Consider the predictor with the highest P-value. If P > SL, go to STEP 4, otherwise go to FIN

STEP 4: Remove the predictor

STEP 5: Fit model without this variable*

Courtesy of Kirill Eremenko

Building A Model



Forward Selection

STEP 1: Select a significance level to enter the model (e.g. SL = 0.05)

STEP 2: Fit all simple regression models $\mathbf{y \sim x_n}$ Select the one with the lowest P-value

STEP 3: Keep this variable and fit all possible models with one extra predictor added to the one(s) you already have

STEP 4: Consider the predictor with the lowest P-value. If P < SL, go to STEP 3, otherwise go to FIN

Courtesy of Kirill Eremenko

Regressions

- Simple Linear Regression

  $y = b_0 + b_1 * x_1$

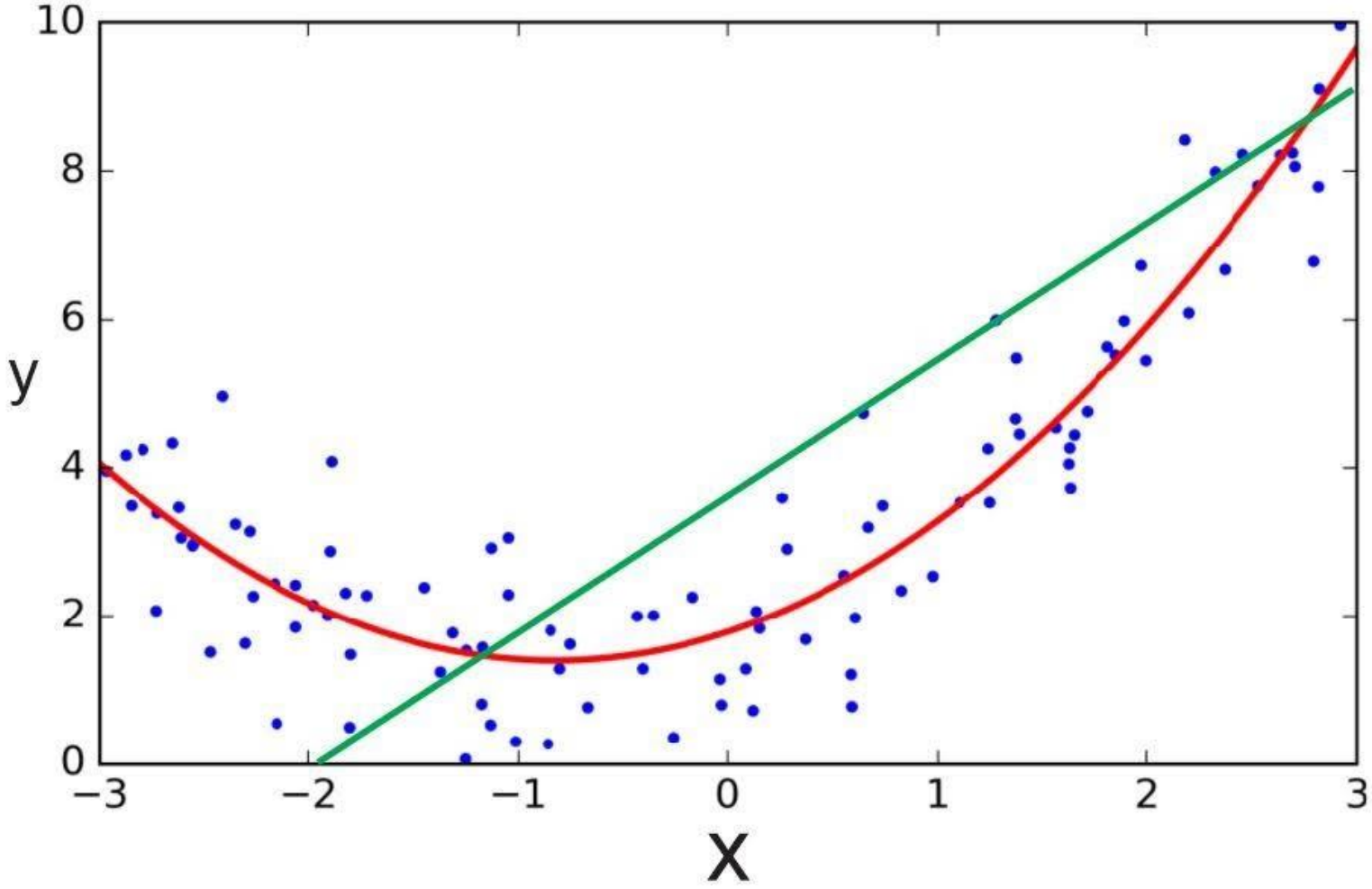
- Multiple Linear Regression

  $y = b_0 + b_1 * x_1 + b_2 * x_2 + b_3 * x_3 + \ldots + b_n * x_n$

- Polynomial Linear Regression

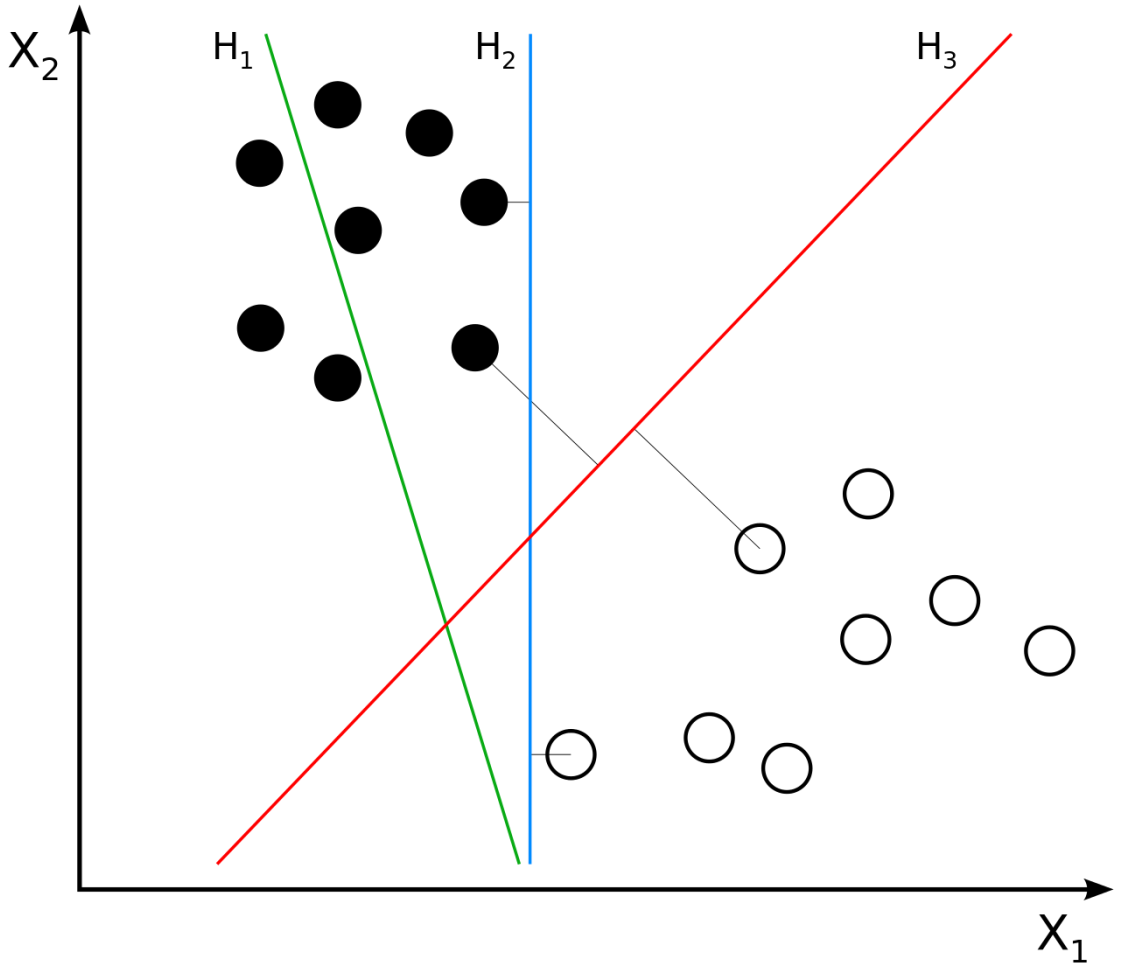
  $y = b_0 + b_1 * x_1 + b_2 * x_1^2 + b_3 * x_1^3 + \ldots + b_n * x_1^n$

Polynomial vs. Linear Regression



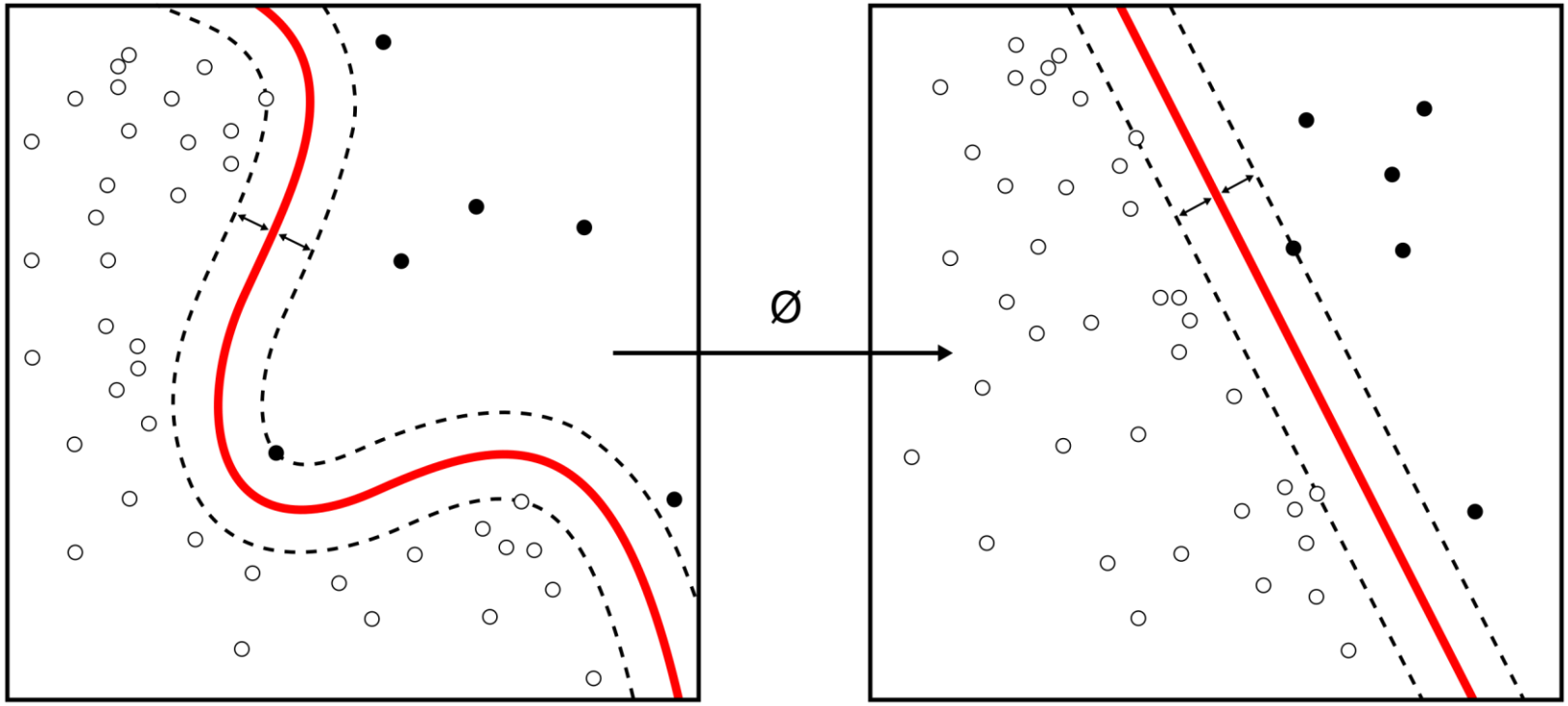
https://harangdev.github.io/machine-learning/5/

Support Vector Regression



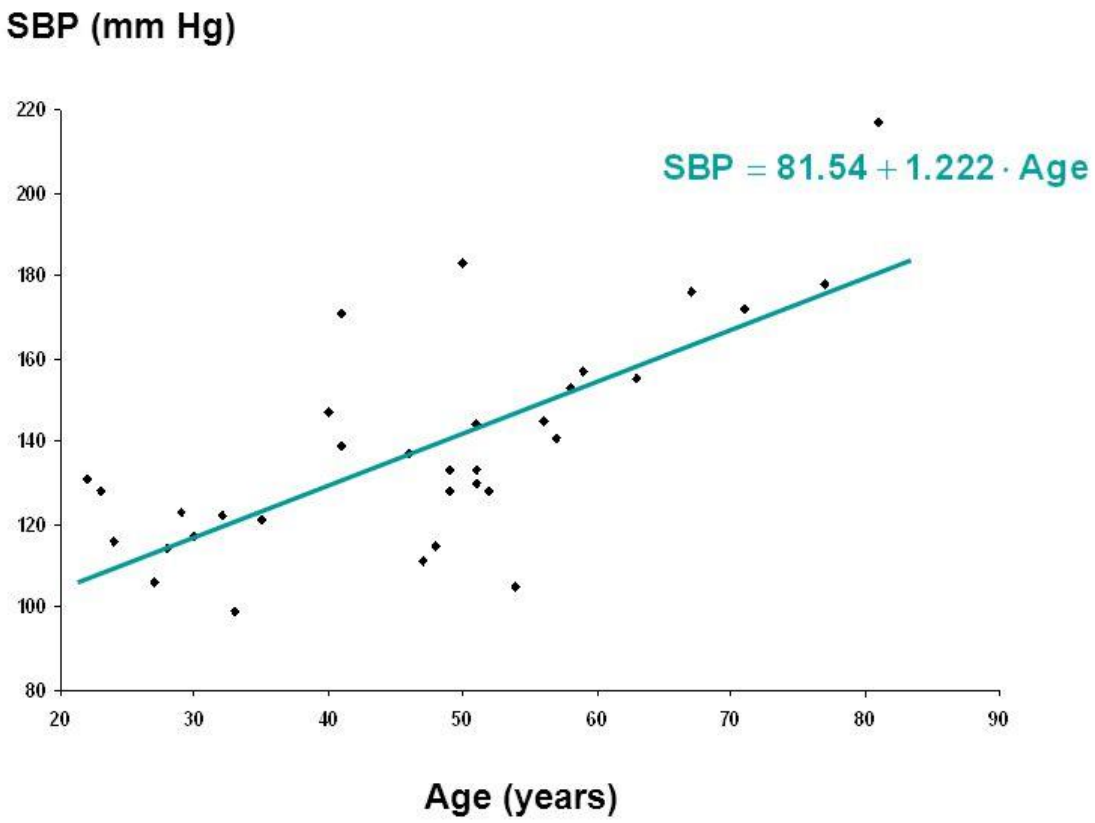
https://en.wikipedia.org/wiki/Support-vector_machine#/media/File:Svm_separating_hyperplanes_(SVG).svg

Support Vector Regression



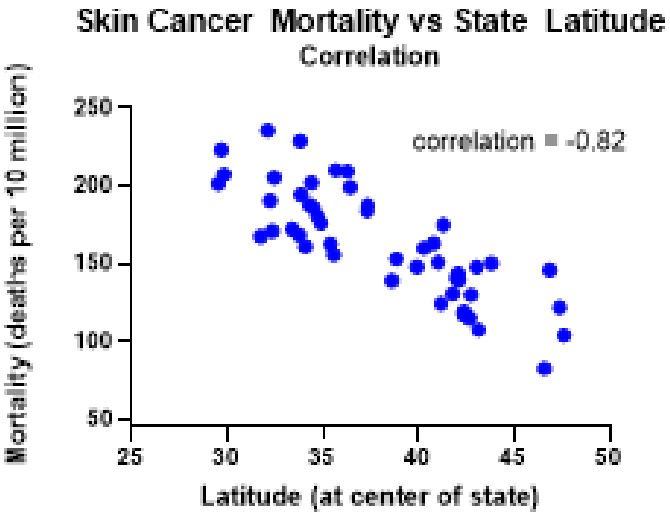
https://en.wikipedia.org/wiki/Support-vector_machine#/media/File:Kernel_Machine.svg

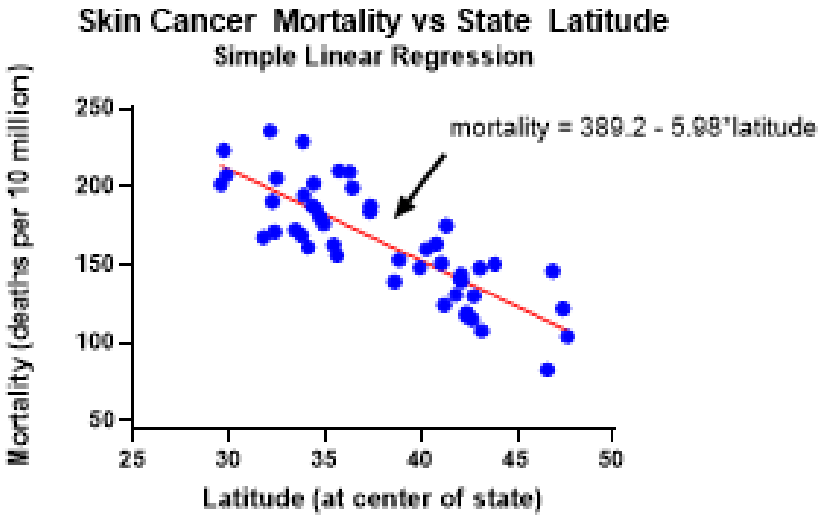
Applications of Regression



adapted from Colton T. Statistics in Medicine. Boston: Little Brown, 1974

Correlation vs. Coefficient





https://www.graphpad.com/support/faq/what-is-the-difference-between-correlation-and-linear-regression/

Summary of Regression Methods

- Widely used in medical research
- Easy implementation
- Good interpretability
- Limited learnability